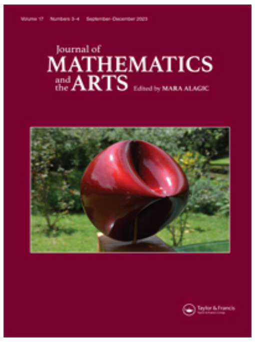

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Journal of Mathematics and the Arts

ISSN: (Print) (Online) Journal homepage: www.tandfonline.com/journals/tmaa20

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To cite this article: Ásgerður Harriss Jóhannesdóttir, Rannveig Björk Thorkelsdóttir & Berglind Gísladóttir (12 Mar 2024): Joining the Math Circus: exploring advanced mathematics through collaborative hands-on activities and performative storytelling, Journal of Mathematics and the Arts, DOI: 10.1080/17513472.2024.2321564

To link to this article: https://doi.org/10.1080/17513472.2024.2321564

Published online: 12 Mar 2024.

JOURNAL OF MATHEMATICS AND THE ARTS
https://doi.org/10.1080/17513472.2024.2321564

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ABSTRACT

This paper details a qualitative study examining the experience and motivation of participants, facilitators, and organizers of the Math Circus, an event hosted by the University of Arkansas Honors College in Fayetteville, Arkansas, as part of the university's 150th anniversary celebrations. The aim of the Math Circus was to entice participants to explore advanced mathematical concepts through collective mathematical art creations and dramatic storytelling of comic tales around the history behind mathematics. The study was carried out in two phases; an autoethnographic case study during the event; and a series of deep, semi-structured interviews with participants, facilitators, and organizers. The study examined pedagogical motivations behind the event's activities and their mathematical content, as well as the experiences of the interviewees. Findings indicate that an interactive and festive event like the Math Circus is likely to encourage confidence to explore mathematical concepts further.

ARTICLE HISTORY

KEYWORDS
Math Circus; drama in education; performative storytelling; creativity; play; mathematics education; hands-on mathematics

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Introduction

This study looks at an innovative mathematics learning space that fused together hands-on collaboration and performative storytelling to explore advanced mathematical ideas. This innovation is the University of Arkansas' Honors College Math Circus; a daylong festive carnival-style event, encompassing an array of mathematics learning attributions. The event was a part of the University of Arkansas' 150th anniversary celebration *Come as You Arkansas.* Additionally, prior to the day of the Math Circus event, the Honors College offered a two-week long Signature Seminar to those interested in exploring the logistics of launching a Math Circus event and designing collaborative mathematical activities for attendees to participate in.

The aim of the study was to cast a light on the educational philosophy and pedagogical goals behind the choices for the event's mathematical activities and to unravel participants' perceptions of their experiences of being invited to explore complex mathematical ideas through hands-on collaborations combined with performative storytelling. Furthermore, the potential of the Math Circus to open up pathways to mathematical intuition through interactive, and playful praxis, is explored.

Theoretical background

To place the Math Circus within a scholarly context of current calls for more creative approaches to mathematics education, this paper considers two strands of theoretical assumptions: Theories of performative storytelling, through drama in education; and theories about the role of creativity in mathematics education. Intermeshed in these threads are considerations of joy, and playful pedagogies, theories around concretiing abstract ideas, and social theories of mathematics education.

Drama and performative storytelling in education

Traditions of learning through drama and storytelling are deeply rooted in many cultures but here the focus is on scholarly prepositions developed in the United Kingdom from the mid-twentieth century into the 21st. Initially, revolving around the educational value of a theatrical product or performance the debate moved towards whether drama in education should be considered a specific school subject, or used as a teaching platform (O'Neill, 2015). Emerging from the *Mantle of the Expert* approach, developed by Dorothy Heathcote and Gavin Bolton in the 70s and 80s (Heathcote & Bolton, 1995), the focus has shifted to pedagogical approaches such as *Process Drama* that relies on interactive collaboration between students and the teacher, in- or out-of-role, within an imagined scenario instead of a final product (Fleming, 2017). Scholars like Jonathan Neelands (1984); Cecily O'Neill (1995); Joe Winston (1998); David Booth (1994); John O'Toole (2016); and Brian Edmiston (2012) have since developed the discourse with their theories around how drama and storytelling benefit a range of educational goals including moral development, language, and other subjects.

Building on previous assumptions, the recent approach places drama in education in context with social and political development and dives deeper into the cross-cultural and social values, as well as its authenticity qualities (Piazzoli, 2014; 2022). Drama in education can empower people to deal with conflicts and social change (Anderson, 2014; 2022; O'Toole, 2016; Thorkelsdóttir, 2018), and the imaginary dimension of drama allows exploration of sensitive topics that fit well within the discourse around the humanizing education (Edmiston & Towler-Evans, 2022; Chemi, 2018; Jennings, 2016; Tomlinson, 2016; Walsh et al., 2017). Some scholars even claim that drama and storytelling in education is simply necessary for holistic education and wellbeing in times of globalization (van de Water, 2021).

Research shows that through storytelling, students are able to utilize a different type of lens to develop the necessary critical and analytical thinking skills to explore mathematical ideas (Arneja & Tyagi, 2020). Stories stimulate thinking and imagination and can extend 'mental accuracy and concentration' to be applied to previous experience and understanding (Reyhanipoor et al., 2022, p. 57). Learning can be seen as something 'that emerges during performative exploration' where the learners 'interpret the actions, events, responses' and 'engage with empathy and conviction in the performative spaces' (Fels & Belliveau, 2008, p. 49). Through drama and storytelling, students may develop their self-expression and can build the confidence and the skills needed to work with others, and drama may also enhance creativity (Thorkelsdóttir & Jónsdóóttir, 2022).

Creativity and play in mathematics education

Creativity has traditionally been considered more an element of the arts than mathematics and science (Bicer, 2021). As it turns out though, 'mathematics is inherently creative and imaginative' (Pound & Lee, 2021, p. 1495). Many believe that mathematical ability is only given to the most gifted, and earlier research assumed mathematical creativity to be an innate power (Riling, 2020). In recent years, creativity and psychological aspects have been considered fundamental for mathematical understanding, and scholars largely agree that creativity entails flexibility to apply critical and analytical thinking to make assumptions from new ideas or imagined scenarios (Robinson & Koshy, 2015; Kettler, Lamb, & Puryear, 2021). This means that creativity entails contemplation and making connections through thinking, rethinking, and stretching the imagination in all aspects of learning and life (Pound & Lee, 2021). Mathematicians value creativity, and most educators know that every student is capable of using creativity at all levels of mathematics (Cilli-Turner et al., 2020). Yet, the image of 'what it means to do mathematics and be engaged in mathematics' is rarely displayed in the mathematics classroom (Pound & Lee, 2021, p. 1495; Cilli-Turner et al., 2020).

The learning benefits of play are solidly recognized in mathematics education and are closely linked to how we conceptualize abstract ideas, and play-based learning environments are considered to benefit students' academic achievements as well as cognitive and social development (Murtagh et al., 2022). However, some scholars are concerned that the complexity of play makes it difficult to define and conceptualize (van Oers, 2013) and measure (Dockett & Perry, 2008). This is because the activity of play is not always playful, especially when it is required for completing a particular task, and in contrast, a playful activity does not always look like play, for example when one dives into a mathematical task and solves it with a playful attitude (Gresalfi et al., 2018). Constantly evolving, current theories of the learning nature of play value its socio-cultural, educational, and therapeutic aspects, viewing it as a multivariable *dynamic system* of learning through prior experience and skills (Bergen, 2014).

Concretizing the abstract

Due to the abstract nature of mathematics, many people are hindered from enjoying its beauty and creative elements (Knorr et al., 2019). Along with the notorious reputation of mathematics being a difficult subject, and the challenge of concretizing the abstract, this makes prospering in mathematics a hard task for some (Kukey et al., 2019). Conceptualizing is critical to mathematical learning as it provides access to appropriate methodology to solve mathematical problems (Hussein, 2022). Interactive, hands-on activities, and mathematical modelling of real objects can enhance learning experiences in mathematics and the sciences by improving conceptualization and knowledge building (Ekwueme et al., 2015; Nurjanah et al., 2021; Weinhandl & Lavicza, 2021). Creativity and equity in mathematics go hand in hand as mathematics naturally carries within it constant questioning and logical reasoning, that are in itself elements of democracy and inclusion (Auckly, 2019), and creative learning environments promote stronger learning outcomes (Kozlowski & Si, 2019). However, administrative pressures to 'complete the curriculum and increase students' pass rates' can be an obstacle to conceptual mathematical teaching, thus, quantity becomes the focus over understanding (Hussein, 2022, p. 60). These obstacles are at the heart of our inquiry as the nature of the examined methods promotes creative learning conditions.

Social mathematics

In times when navigating the mess of global inequities, political turmoil, pandemics, and informational chaos, the skills of applying critical and analytical thinking become essential (Abel & McQueen, 2021; Karami & Parra-Martinez, 2021). Providing relatable and playful ways for students to engage with mathematics may assist in developing such skills (Segarra et al., 2018).

The call for humanizing pedagogies in education, including mathematics, has become stronger recently and 'if doing mathematics is like navigating a sailboat, then human desires are the winds that power the sails, and virtues are the qualities of character that sailing builds – mindfulness, attention, and harmony with the wind' (Su, 2020, p. 11). Drawing on Freire's ideas (1970), Yeh and Otis (2019) find that humanizing pedagogy in education, including mathematics, can foster social justice and equality. They also see mathematics as an instrument to revamp social power patterns which is in line with Chen and Horn's (2022) call for critical bifocality.

There are strong indicators that cognitive flexibility increases when learner autonomy is high (Orakci, 2021). This underpins the importance of learner autonomy for mathematics education (Horn, 2017) but exercising that agency in traditional settings of mathematics education is rare, and a playful approach is even more so (Jasien, 2020). Learner autonomy can also affect social justice as it promotes confidence to apply critical thinking, discuss and debate ideas, and take action (Sachdeva, 2019).

Methodology

This study was qualitative, embracing the *Big Q* methodology, where qualitative data, values, and practices are embedded in a qualitative paradigm (Braun & Clarke, 2013; 2020; 2021; 2022). The inquiry falls under phenomenological study as it illustrates shared meanings for a group of individuals who have experienced a certain phenomenon (Creswell & Poth, 2018). The phenomenological approach focuses on identifying common aspects of the research participants' lived experiences in order to concentrate on the universal essence of that particular shared experience (Creswell & Poth, 2018).

The research was twofold; an *autoethnographic* case study, where the researcher attended the Math Circus and used self-reflection and writing to analyze the experience of participating in the different activities. These self-reflections were then placed within a wider context of data collected through *semi-structured interviews* with participants, facilitators, and organizers of the Math Circus, who reflected on their experiences. Some of the interviewees had also participated in the preceding Math Circus signature seminar. The seminar's discovery-based design gave students opportunities to analyze what a successful implementation of a Math Circus entailed, as well as experimenting with creating collaborative mathematical activities to offer at the event.

Autoethnography is increasingly being implemented in qualitative research but there are levels of academic suspicion against it due to a lack of measurable quantitative research methods (Holt, 2003). Overall, autoethnography is 'accepted as a research method capable of powerful examinations of the relationships between self and other from the perspective of self ' (Starr, 2010, p. 7). Some scholars call for a sharp division between *evocative autoethnography*, where the goal is to evoke the reader's emotions rather than to remain objective, and *analytic autoethnography,* where theoretical assumptions are derived from empirical data, others consider the multi-dimensional aspect of autoethnography its core value (Le Roux, 2016). This study embraces the flexibility and creativity rooted in the method at the same time as researchers are aware that rigour is essential to accomplish sincere results (Cooper & Lilyea, 2022).

Research participants, data collection, and management

To obtain data beyond autoethnographic notes, the researcher approached Chaim Goodman-Strauss, the mastermind behind the Math Circus, for an interview regarding his goals and experience related to the event. Goodman-Strauss offered to contact others who had been involved, to ask if they were also interested in sharing their experiences. Through these connections, deep, open-ended interviews were conducted with 15 individuals who all were involved in the Math Circus in one way or another. The age of individuals ranges from 19 to 68 years old; 9 identified as male, and 6 as female; 80 percent as Caucasian; 20 percent as people of colour. A total of 95 percent of the interviewees were university-educated or were pursuing a university degree at the time. The interviewees included Goodman-Strauss and his assistant, the Town Crier, Honors College administrative staff, event participants, educators, and university students who took part in the Math Circus activities, and students who both participated in the preludial signature seminar and helped facilitate activities on the day of the Math Circus.

Prior to the interviews, interviewees were presented with an information sheet explaining the purpose and methodology of the study. Informed consent was obtained ahead of time, and the interviews were conducted either in person or over secure online meeting platforms. Once collected, all data was encrypted, password protected, and securely stored. Interviewees were given the option of anonymity, or to have their names included in the write-up. Each interview was audio recorded and lasted on average from 45 to 60 min. The recordings were transcribed and thematically analyzed. Once processing and relevant publications are completed, all data will be securely deleted.

The general request for commitment was around 45 min but interviewees had the choice to extend the time or cut it short. In most cases, the interviewees got really engaged in the conversation and freely extended the interview time. One follow-up interview was conducted with Goodman-Strauss to get a deeper understanding of the mathematics behind the activities. The interviewees did not receive any payments or other compensations for their participation.

Data analysis

Autoethnographic, self-reflective notes from the researcher's observation and participation, and data obtained from the interviews, were thematically analyzed in line with Braun & Clarke's suggestions (2013; 2020; 2021; 2022). The coding and analytic work started right from the first interview, aiming to identify and develop patterns, themes and categories, within the interviewees' experiences. It is important to avoid determining the themes too early in the process but instead establish them where the data lined up with the researchers' background knowledge, experiences, and conceptual and theoretical understandings (Braun & Clarke, 2022).

The study takes into account Braun and Clarke's methods of reflexive thematic analysis that embrace the profusion of methods where the researchers' theoretical and methodological stances are acknowledged through contextual consideration and principles (Braun & Clarke, 2021). In our case, the primary researcher's background in drama in education and long-term involvement in a community of mathematicians dedicated to the creative communication of mathematics shapes her theoretical assumptions that drama and storytelling can benefit the communication of abstract mathematical concepts. Combining hands-on mathematical activities and dramatic storytelling further contributes to the assumption that the Math Circus event compasses multidimensional educational values.

The researcher's role in the autoethnographic research was partially one of an insider as she had previously participated in Goodman-Strauss' mathematics construction events. By participating in the Math Circus, the researcher was aware that her presence could potentially affect the research. The researcher made efforts not to direct actions or lead discussions beyond what would be expected of any other participant.

Analysis and discussion

This section is divided into three parts. The first segment could be viewed as a playbill where the main actor, director, and producer of the Math Circus are introduced, the plot is described through the researcher's ethnographic lens and finally, the mathematical content of the activities is explained. Preluded by the main themes and patterns identified in the data, the second, and third parts provide a critique of the Math Circus' performance through two-stranded theoretical assumptions; performative storytelling in education, and learning theories of mathematics.

The playbill

The main actor. In addition to advanced mathematical content, the Math Circus included a *Town Crier* whose role was played by Dr. Daniel Levine, a professor of Classical Studies at the University of Arkansas. Through performative storytelling and improvization Levine told comic tales of the historical background of the mathematics content, engaged with the crowd, and through dramatic performance gently ushered people to participate. Levine has a passion for making education relevant for students so they can create new understandings. He regularly applies drama and storytelling in his teaching of the classics to create an entertaining platform for his students to relate to ancient history.

Director (and actor). The Ringmaster and mastermind behind the Math Circus, University of Arkansas' mathematics professor and artist Chaim Goodman-Strauss, was both a director and an actor as he designed most of the collaborative activities and also played a part in constructing the objects. Goodman-Strauss has worked on demystifying mathematical concepts through interactive participation for over twenty years and has developed community events exploring advanced mathematical topics, including creating mathematical sculptures in partnership with local artist Eugene Sargent who also assisted with the Math Circus. Their sculptures have been assembled in collaboration with participants in the biennial Gathering 4 Gardner conference in Atlanta, GA, in honour of the long-time Scientific American columnist Martin Gardner who was not trained as a mathematician but devoted his life to communicating mathematics through intriguing and playful games and puzzles (G4G, 2014).

The producer. The Honors College hosted the event, and their director of curriculum and learning played the role of the Math Circus producer. He was not particularly visible during the event, but he played a complex role in making sure every detail was in place to guarantee that the event would go smoothly. During the weeks leading up to the event, the producer juggled many different balls in collaboration with the Honors College's administrative team who all contributed to the smooth delivery of a campus-wide event. Their work included securing funding, predicting costs, anticipating sufficient supplies and materials for the event, recruiting and organizing volunteers, providing hydration and food for volunteers, accommodating pandemic safety restrictions, preparing for the final installation of hyperbolic sculptures constructed during the event, and making sure that the Math Circus would be executed smoothly and safely for all.

The plot; Ethnographic observations. By the time, I arrived at the Math Circus, activity stations had been set up and groups of people were already collaborating to create mathematical structures. The Honors College courtyard was filled with life and action and in every direction I looked, mathematical constructions were being raised. Some people were playing with mathematical objects, others were busily tessellating the sidewalks with brightly coloured chalk creating mesmerizing shapes and patterns. I also noticed groups of children playing with brightly coloured foam spheres, and Curvahedra[1] models. As COVID was still around, everyone was masked but I could identify smiling eyes, I heard laughter, and I saw very focussed body gestures, even confused, and a few that appeared

Figure 1. Bamboo Star Creations. Note. Taken by Hiba Tahir. September 10th, 2021.

somewhat surprised. As I stopped for a moment, I noticed a strangely dressed person walking around the space wearing a top hat and a colourful bow- tie, carrying a megaphone. I quickly realized that this had to be the Town Crier as within a moment he started telling the tale of Euclid insisting to the Greek King of Egypt that there were no royal roads to Geometry. The Town Crier's body language was very dramatic and the comedic projection of his voice made the audience laugh. From this first delightful impression, I decided to join the Math Circus.

Bamboo stars. I started by helping out with a large bamboo star construction. I was nervous at first to join a group of strangers on my own but those who were already working on the stars welcomed me and showed me how to slide 6-foot-long bamboo rods through a jig comprised of an icosahedron, and then tie the rods together on one end. The first steps were easy but things got more complicated as the work developed. I noticed excitement amongst the participants and how the dynamics changed when mistakes were discovered and we needed to step back to figure out why things failed. I did not feel that anyone got blamed when we hit obstacles and discovered errors along the way, and the collaboration got closer as the work moved on. Communicating with other participants was mostly smooth, although I sometimes felt reluctant to take the lead, unless I was sure my ideas would work. I noticed another group working on bamboo stars where a few young people seemed excited to create the stars as they worked fast and seemed to know what they needed to do. Collaboration was at the heart of this activity as completing the stars on one's own would not be impossible but quite a physical challenge. I was curious to discover the mathematical elements of the stars. I had noticed the pentagonal shapes at the core and that there were 5 bamboo rods in each group threaded through the jig but I did not understand what the main mathematical properties were. I also tried to envision what the stars' centres might look like if I could slice through them once all the bamboo strands were inserted. My main challenge was to think ahead about the properties of the bamboo weaving and to resist tying the ends together too soon, which would lock the jigs inside (Figures 1–3).

Tessellations. Next, I participated in tessellating the concrete pavements with laser-cut stencils and chalk. The stencils were polygons filled with smaller designs such as circles and triangles. This activity both offered collaboration and individual work. I observed groups

Figure 2. Bamboo Star Collaboration. Note. Taken by Hiba Tahir. September 10th, 2021.

Figure 3. Bamboo Star Weaving. Note. Taken by Russell Cothren, University of Arkansas – University Relations. September 10th, 2021.

of children and adults collaborating, as well as individuals enjoying the work in solitude. I was compelled by how beautifully the negative shapes turned out on the pavement. The chalk was brightly coloured, which made the shapes pop out and when repeatedly placed next to each other they tessellated the surface. I found this symbolic of the spirit of the event; the individual shapes were nice but when they came together, they transformed the space. The same applied to the bamboo stars, where single bamboo rods were combined into groups to create interesting shapes. The challenging aspect of this activity for me was mainly physical as kneeling down in the hot sun was hard. I enjoyed anticipating what the repeated patterns would look like once coloured on the pavement. Sometimes the results differed from what I had anticipated (Figure 4).

Figure 4. Tessellations in progress. Note. Taken by Ásgerður H Jóhannesdóttir. September 10th, 2021.

Figure 5. Curvahedra Play a the giant Curvahedra sculpture. Note. Taken by Ásgerður H Jóhannesdóttir. September 10th, 2021.

Curvahedra play. After a short break, I visited the Curvahedra booth, located under the giant steel Curvahedra[2] sculpture. I observed children playing with different Curvahedra models and a small group of people talking to the facilitator about the properties of the models. Later, I noticed the Dean of the Honors College was holding office hours under the sculpture (Figure 5).

Hyperbolic sculpture. My last activity to participate in was the rainbow-coloured hyperbolic structures. This activity was directed by the Math Circus mastermind professor Chaim Goodman-Strauss. Assisting with the work and installation was local sculptor Eugene Sargent. Participants were applying superglue to attach the short ends of long strips

Figure 6. Hyperbolic sculpture being made. Note. Taken by Ásgerður H Jóhannesdóttir. September 10th, 2021.

of laser-cut foam material when I arrived, and I could see that a lot of material had already been prepared that way. The strips' sides were attached together through an interlocking 'teeth' system of different numbers of teeth on each side. By attaching the interlocking teeth, the strips start to form a hyperbolic surface that starts as a cone on one end but starts frilling like lettuce leaves on the other end. It was satisfying to feel in my hands how the surface of a flat material started to change. To not only see but feel how the different number of 'teeth' on each side controlled the flat material into a three-dimensional shape was very meaningful to me and the rainbow colours gave the activity a sense of joy. Through the hyperbolic construction, I was able to make a connection between how a different number of connecting teeth on each side of a strip control what type of hyperbolic surface appears. I now also feel more confident to play with that type of manipulation and feel that I can better envision in my head what it would look like if the number of connecting teeth changed (Figure 6).

Moiré Patterns. During another break, I wandered inside the Honors College foyer where the hyperbolic sculpture would later be installed over the stairwell. As I went through, I noticed a curious mathematical item on display. I recognized that this item was built on Moiré patterns; an 'interference pattern' that is created by placing similar but slightly different patterns on top of another. I had recently seen some artwork built on these patterns so had an idea how they worked but did not engage with the item as it felt like more of an object to marvel at than a hands-on activity.

Polyhedron Structure. On my way back out to re-join the hyperbolic sculpture construction I noticed a couple of people building a polyhedron out of plastic tubing. It looked like it was slightly unstable but it had an appealing look to it. This appeared more like an individual's work than a collaboration. I later discovered that this was a student's experiment to make polyhedra out of that material (Figure 7).

Genuine Spheres. I noticed a group of children playing with curiously looking, brightly coloured spherical things made from a similar material as the hyperbolic construction. I saw small children throwing the spheres in the air like they were juggling, and others using it like a crown on their head. I was not able to find anyone who knew what these were

Figure 7. Polyhedron construction. Note. Taken by Ásgerður H Jóhannesdóttir. September 10th, 2021.

Figure 8. Genuine Spheres, hyperbolic sculpture materials (notice the 'teeth). Note. Taken by Ásgerður H Jóhannesdóttir. September 10th, 2021.

apart from someone telling me that no matter how you would attach the strips you would end up with a spherical shape. Later on, I noticed these spheres being interspersed in the hyperbolic sculpture (Figure 8).

Hyperbolic sculpture installation. The last activity I observed was the installation of the hyperbolic sculptures. It took several people to hold them up and hang them above the stairs in the Honors College foyer. I was impressed with how the facilitators used long wooden sticks to bring the structures to the high ceiling. They had previously installed hooks in the ceiling as well as strings and hooks to the structure itself. There was a sense of urgency to get the structures installed before the event would formally end. There were still many people helping out and there was both a joint excitement and a slight worry that the system would not work. Everything worked out and the three pieces were safely installed in a matter of minutes. I was getting very tired and could feel the effect of the sun and heat. I worried that something would go wrong and the sculptures would crash into the floor, ruining the hard work of so many people. However, it was soon clear to me

Figure 9. Hyperbolic Sculpture being installed in the UA Honors College foyer. Note. Taken by Kendall Curlee. September 10th, 2021.

Figure 10. Hyperbolic Sculpture and metal Curvahedra in the UA Honors College foyer. Note. Taken by Kendall Curlee. September 10th, 2021.

how well prepared and confident the facilitators were when they guided the team to carry the structures inside and then to collaborate with the final installation into the ceiling. The pieces will stay in this place for a while and serve as a monument for everyone who participated in the Math Circus. Participating in the Math Circus was a great pleasure for me, and I enjoy visiting the hyperbolic sculptures in the Honors College foyer whenever I have an errand on campus, and I look forward to any similar events I may be able to attend in the future (Figures 9 and 10).

Town Crier. The Town Crier walked around the area, telling funny stories of the ancient Greeks. His performance elevated my experience, making me think about the ancient Greeks cracking mathematics, coming up with theorems to understand the world. I enjoyed the Town Crier's appearance which was subtly dramatic with the tophat and bowtie, and his whole performance radiated amusement. His comedic narrative about the ancient Greeks and Romans connected my mind to the background of what was being worked on right

Figure 11. The Town Crier. Note. Taken by Ásgerður H Jóhannesdóttir, September 10th, 2021.

there and then, and made me think about how little we, or at least I, normally think about the history of mathematics and how ancient ideology has shaped current mathematical understandings. I noticed the Town Crier catching the attention of many of the participants. Many people laughed but some did not look like they were paying any attention to him, but then suddenly they would snicker at the comedic stories but keep their workflow going. This indicated to me that the performance was being observed and heard without it being socially necessary to demonstrate it. I also noticed that the Town Crier would sometimes interact with passers-by and gently invite them to join the activities or ask questions. Through the Town Crier's performance, I felt a connection with other participants even though I was not directly interacting with them. We were all there together at this very moment, figuring out a joint task (Figure 11).

Math Circus mathematics

Bamboo stars. Early in his career, Professor Goodman-Strauss built an icosahedral structure consisting of rhombuses connected in points of three, and five strands (Figure 12); a *rhombic triacontahedron*.[3] Later he came upon some giant bamboo star constructions in Austin's Botanical Gardens. At the time, Goodman-Strauss had no idea that the bamboo star constructions were made by Akio Hizume, a Japanese architect and mathematical artist who developed the Giant '*Star Cage*' Bamboo Stars (Hizume, n.d.). Later, the pieces came together when the two met at a *Gathering for Gardner* conference in Atlanta, GA. Additionally, in the 60s, John Kostick (Kostick, n.d.) developed several systems of those structures. Immediately, Goodman-Strauss recognized the core structure of the giant bamboo stars as that icosahedral structure he'd been exploring. His wife suggested that perhaps he could recreate these giant starts with his students as a fun activity. Goodman-Strauss pursued the idea, designed a convenient jig to guide the construction (see pictures below), and as a result has been facilitating collective constructions of them with students and different community groups for over twenty years (Goodman-Strauss, 2021). The technique creates

Figure 12. Modelling the bamboo star geometry. Note. A model made by Chaim Goodman-Strauss showing the geometry behind the weaving patterns of the bamboo stars. The paper strips show where the bamboo goes around a rhombic triacontahedron, a polyhedron with 30 rhombic faces, and icosahedral symmetry (the red ZomeTool). Photo by Edmund Harriss. 2022.

an *icosahedral* star woven out of 30 six-foot long bamboo sticks divided into groups of fives. Each group of fives is passed 'through two opposite faces of a *dodecahedron*, as a pentagon rotated slight[ly] with respect to the pentagons they pass through. The 30 lengths weave together in the middle, needing no other support' (Harriss, 2011).

Geometric tessellations. The Geometric Tessellations (Figures 13–15) were designed and made by students of the preludial Math Circus Signature Seminar, and consisted of geometric shapes which could tile the plane. The designs, inspired by *Penrose tilings*[4] and *Islamic geometric tiling patterns*[5] were made of basic geometric shapes such as hexagons, squares, and equilateral triangles. These were then filled with smaller geometric designs to make them more aesthetic and interesting.

Additionally, the Seminar students created a selection of stickers with information and images linked to mathematical concepts to be given to participants at the Math Circus (Hogue, 2021).

Curvahedra play. The Curvahedra Play activity took place underneath a 12-foot-diameter steel Curvahedra[6] located in the Honors College Courtyard (UA Honors College, 2022). The Curvahedra system is a geometry system related to polyhedra consisting of thin, flat, laser-cut mylar pieces that lock together to open endless possibilities for creating polyhedra with curved edges rather than flat polygons. Each piece has a central point corresponding to polyhedral vertices where three, four, five, or six curves branch out. The angle between the curves, combined with the loops they form, forces them to bend into different shapes when the pieces get attached to each other. As Harriss (2020) describes, the icosahedral Curvahedron sphere is made from 12 flat 5-branch pieces creating 20 individual curved triangles. The sum of the triangles' inside angle-sum is greater than 180°,

Figure 13. Tessellations in action. Note. Taken by Grace Li. September 10th, 2021.

Figure 14. Tessellations in action. Note. Taken by Grace Li. September 10th, 2021.

Figure 15. Tessellations in action. Note. Taken by Grace Li. September 10th, 2021.

which creates a positive curvature. In this context, Harriss explains how Gaussian curvature details three different types of curvature: positive for a sphere, zero for a flat surface, and negative for a saddle, and that the number of branches around a loop and the angles at each vertex of a Curvahedra piece determines the curvature of the region it surrounds, with the relationship given by the Gauss-Bonnet theorem. Adding extra edges to a loop or replacing a piece with one with more branches decrease the Gaussian curvature (Harriss, 2020).

Hyperbolic sculpture. The giant hyperbolic sculpture is constructed from laser-cut foam materials (similar to children's playmats) which have interlocking 'teeth' shapes along their sides. This means that the strips can be connected without glue or other adhesives (though, at the Math Circus, glue was used to connect to form lengths of the strips to better secure the structure). The number of 'teeth' on one side of the strip is less than on the other, creating an increase resulting in a *hyperbolic surface* that starts to 'frill' like lettuce or kale leaves (Hogue, 2021). The hyperbolic surface or geometry of the sculpture falls under *non-Euclidean geometry,* which means that it does not abide by Euclid's fifth axiom, the 'parallel' (Drach & Schwartz, 2020, p. 61). Two notable negations on Euclid's parallel axiom were placed by Playfair (b. 1748 - d. 1819), and Lobachevsky (b. 1792 - d. 1856) (Lewis, 1920). Playfair stated that more than one parallel line can pass through a point, meaning that parallel lines could intersect at some point (Brown et al., 2019). Lobachevsky claimed that a specific line and point that are not on the line means an unlimited number of lines going through the point parallel to the given line (Ravindran, 2007). Lobachevsky's negation as well as the works of Bolyai, and Gauss is the foundation of hyperbolic geometry where parallel lines don't meet but the angle sum of a triangle can be less than 180 degrees (Ravindran, 2007). Non-Euclidean geometry became a popular research topic during the first half of the nineteenth century through eager attempts to grasp Euclid's five postulates and then became a prominent part of mathematics and physics research throughout the mid-twentieth century (Cannon et al., 1997) (Figure 16).

Figure 16. Hyperbolic sculpture in the making. Note. Taken by Ásgerður H Jóhannesdóttir. September 10th, 2021

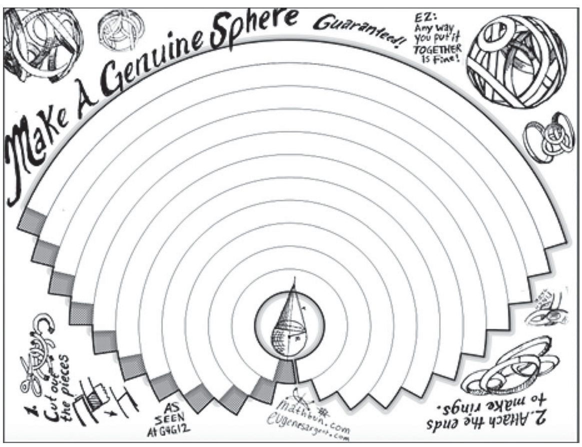

Figure 17. Genuine Sphere. Note. Map of a 'Genuine Sphere'. Design by Eugene Sargent. N.d.

Genuine spheres. Another activity was the *Genuine Sphere,* made from laser-cut foam strips like analogues of straight lines, showing the shortest distance paths (to walk on the Geodesic surface). They have interlocking teeth on each end and no matter how you connect them you will end up with a 'genuine sphere' (Goodman-Strauss, 2022). Several of the participants' made Genuine Spheres became parts of the Rainbow sculpture (Figures 17–19).

Figure 18. Child playing with making a 'Genuine Sphere'. Note. Taken by Hiba Tahir.September 10th, 2021.

Figure 19. Rainbow Sculpture and Genuine Spheres. Note. Taken by Whit Pruitt, University of Arkansas – University Relations. 2021.

Themes and patterns in the data

The following section details the main themes and overlapping patterns identified in the data. The themes are divided into two strands: Themes around the performative storytelling parts of the event that include *invitation to engage, social coherence, relevance & focus, joy, and drama and storytelling for mathematics learning;* And pedagogical themes including *freedom to explore, creativity, and playfulness, social dynamics and cultural relevance,* and *joy.*

Math Circus performative storytelling

> Welcome to the Math Circus. the one and only, unique, singular, exclusive, inimitable, matchless, rare, and one of a kind! it's a *rara avis*, if you catch my latin drift. **See** the negative curvature of hyperbolic geometry! **Experience** the foundations of logic to the shape of the universe! **Weave** a bamboo star! **Marvel** at mathematical sculptures, and make some yourself! 2, 4, 6, 8 … It's so fun to calculate! 1, 2, 3, 4 … Crunching numbers makes you score. 3, 6, 9, 12 … Into numbers we shall delve. 4, 8, 12, 16 … Math can be a trampoline. 5, 10, 15,

> 20 … Mathematics joys are fun and plenty 6, 12, 18, 24 … Make a star and come back for more. 8,16, 24, 32 … Learn geometry, and I mean you! 9. 18, 27, 36 … Math can teach you fancy tricks. 10, 20, 30, 40 … Glory, glory, glory, glory
>
> (Daniel B. Levine, 2021).

Theme 1a: Invitation to engage

Goodman-Strauss aimed for performative storytelling to bring joyful dynamics to the Math Circus and emphasize its circus element. Additionally, he hoped for the Town Crier's performance to enhance people's ability to make connections and invite participants to explore the mathematical activities. Preparing for the Town Crier performance, Levine took on an extensive research of mathematics ideas and terms that he incorporated into his script. His role was both scripted and improvised and when writing the script Levine drew on his previous experiences of facilitating mathematics festivals with Goodman-Strauss. His philosophy was to catch positive attention by exaggeration and going beyond his regular role as a college professor. During the event Levine used the script as a framework and then improvised where the opportunities appeared. When he attracted attention, he would sometimes break the fourth wall to interact with the audience and gently invite them to engage in the activities. He would make eye contact with the audience, respond to them, and address some of them while still in role. Breakthroughs for Levine were the interdisciplinary aspects of the Math Circus and the levels of collaboration across the fields. He felt that his role was appreciated and that he had space to perform his role towards his best skills. Not to prove a theorem but to make sure that participants would know why learning mathematics is a positive thing.

There were patterns in how the attendees observed the Town Crier's performative storytelling to spark interest and extend an invitation to engage in the activities. The children enjoyed listening to the humorous tales of mathematics history and were encouraged by the gentle enticement to participate in the constructions. Willard Keirn, a local middle school mathematics teacher, told us that his students thought the Town Crier's dramatic performance was effective in catching their attention and interest to engage in the activities. The Math Circus learning environment was quite Piagetian as it offered an opportunity to learn without being taught (Smolucha & Smolucha, 2021), which was enhanced by the imaginary aspect of performative storytelling to link historic aspects to the current moment. Our assumption from this theme is that through performative storytelling an invitation to explore complex ideas can be extended to diverse groups who may not have much experience or confidence in a topic (Quiroga, 2015).

Theme 1b: Social aspects

The Town Crier brought a focus point and a positive energy to the event according to our interviewees. They felt that the performance created a perception of social coherence as even though they were not always actively interacting with each other, the performance brought their attention to other participants and a feeling that they were doing something together. One parent said that the performance was a unifying experience and that the Town Crier brought energy to the collaborative aspect of the work, as well as gently making sure that everyone was working together. The performance offered a choice to either idly listen or to engage and explore further. This theme indicates that the performative storytelling element can extend engagement and collaboration between different groups as

Quiroga (2015) pointed out. Keirn noticed different social dynamics in his student group where through the performances and activities social structures shifted and students who normally did not interact with each other became peers at the Math Circus. This is in line with Chen and Horn's (2022) call for learning environments to create space to allow for less visible inequitable social power structures to be dispersed and reorganized in healthier ways.

Theme 1c: Relevance and focus

In his role, Levine made connections between ancient Greek mathematics history and modern society. He analyzed why mathematics should be studied today and how it relates to our daily lives. The Town Crier's stories brought relevance between the event and the history of the concepts that were at the core of the activities. Interviewees said the relative content of the Town Crier's speech brought together awareness of the timeline and history of the mathematics connecting the now to ancient history with names that may not be very relative to the participants' everyday life. Physically, the Town Crier reminded everyone that they were a part of this playful thing that was going on.

Sargent thought that Levine's classical studies background worked really well to connect the activities and the history behind them to the moment in a humorous and human way. He found that the performance tied the whole experience together, and the humour broadened the appeal to every person. Sargent also considers the Town Crier to be a spokesman for mathematics which he brought to an everyday perspective with a radiating humour.

Attendees agreed that the performative storytelling brought focus as well as relevance to their experience of participating in the activities, which is in line with the conclusion of Reyhanipoor et al. (2022) that performative storytelling can bring learning focus and concentration to be applied to previous experience. Cognitively guided instruction (CGI) requires connecting new mathematical concepts to students' previous experiences and existing knowledge through active concretization of abstract concepts (Carpenter et al., 1999/2015). This is exactly what the Math Circus achieves. It takes abstract mathematics concepts that most of us are not used to observing, and allows us to participate and collaborate in concretizing them. The collaboration aspect of the Match Circus' mathematical activities elevated the impact of simply observing ready-made mathematical models, which aligns with Boaler and Humphreys (2005) statement that mathematical learning occurs when new ideas can be connected to students' prior understanding.

Theme 1d: Joy

The Town Crier's performance brought energy and lightness to the otherwise serious mathematics work, connected the activities to the history of mathematics with a humorous approach, and created a focus point beyond the collaboration. Interviewees noticed joyful reactions to the Town Crier's role, and that he was able to improvise and read the audience to gently usher participation without confronting or embarrassing anyone. According to Keirn, his students were amused by the Town Crier's comical attire of a bow tie and a top hat and enjoyed the comedic narration which included the history of mathematics which they thought brought an overall sense of joy to their experience. Many students commented on the role of the town crier in their reflections. They enjoyed the comedic aspect and the fact that connections to what they were doing were being presented to them through the performance without a direct demand for them to engage specifically in the performance, but instead just being able to enjoy the storytelling whilst they worked on figuring out the challenges of the mathematical work. Keirn noticed that the Math Circus activities really encouraged collaboration amongst his students, and at the same time allowed for creativity through the process of trial and error in figuring out solutions to the challenges. Sargent noticed visible joy amongst the participants over the performance that seemed to appeal to attendees' curiosity. These findings are in line with Parks's findings that amongst other methods joy in mathematics can be generated by offering meaningful tasks to be explored through social interaction and material engagement (Parks, 2020).

Theme 1e: Drama and storytelling for mathematics learning

Knowledge is co-constructed through dialogue and story-sharing that brings out 'the centrality of context, communities, histories, relationships, interactions, and connections in learning' (Yeh et al., 2021, p. 90). Goodman-Strauss is positive about using drama to teach mathematics, and regularly implements elements of drama in his university mathematics teaching, but has concerns that a story could be told without making proper mathematical connections. He sees drama/storytelling as an anchor in mathematics education like 'if there is a bit of tack you make a connection and can start exploring more'.

Professor Levine is convinced that drama and storytelling are an empowering medium to build confidence and expand students' perspectives. Although not guaranteed, he believes that drama and storytelling are capable of shifting students' common self-deprecating narratives. Through drama, Levine attempts to positively widen his students' scope towards the view that 'learning is a beautiful thing and that it makes sense'.

Levine is aware that mathematics education means the struggle for some as acquiring mathematics takes patience. This struggle goes back to antiquity as even Euclid was not able to show the Greek king of Egypt any shortcuts to learning mathematics. Therefore, Levine thinks that incorporating drama and storytelling in mathematics education is an important way to provide context to abstract ideas by shifting from the form, digging deeper into the concept, and placing it in social, historical, and philosophical contexts so that it makes more sense.

The most enjoyable part of teaching for Levine is to get reactions from students so he actively tries out language to spark their interest and curiosity. He utilizes drama to bring positive energy to a subject and enjoys getting reactions from people when enthusiasm is shared. In his teaching Levine uses drama to bring ancient texts to life, and through elements of Greek tragedy and comedy he makes historical and social connections with current social affairs.

Sargent thinks that 'drama and storytelling is exactly what math education needs'. As a 'deep and difficult subject that can get overwhelming quickly, mathematics is the language of science and needs to be understood as such'. Storytelling, in Sargent's mind, is a useful vehicle to 'explore complicated terminology that can throw people off and result in loss of understanding'. In a tech-powered world, mathematical skills are important but for many the barriers to mathematical avenues like computer programming, are higher than they need to be, and drama and storytelling are important ways of 'helping it stick in the mind, lower barriers, and spark the interest in mathematics of someone who would not have thought about it before'.

A home-schooling mother who attended the Math Circus with her 7- and 12-year old children, incorporates play in her approach rather than drama and storytelling, though the children rely on stories to deal with mathematical tasks. The narrative helps them connect with their own experiences and put problems into different perspectives. She feels that the narration engages a different part of their brains and suddenly they see the concept in a different context. Narration helps them to connect the abstraction to their own experience and prior understandings. Keirn has used drama and storytelling to teach mathematics and thinks it is very important to incorporate drama and storytelling into general mathematics education to create links between the abstract and students' experience.

The Math Circus and mathematics education

Beyond the entertaining aspects of the Math Circus was a deeply thoughtful pedagogy to be discovered, designed to utilize the joyful festivities to spark interest through concretization of mathematical concepts and performative storytelling. The main goal was to create a joyful moment playing with complex mathematical ideas which then could be used as a reference during future mathematical struggles. Another aim of the event was to crack open the often carefully guarded gates to the creative beauty of mathematics. Furthermore, in the words of Goodman-Strauss, 'the rationale behind the collective structures lies within the fact that by materializing the abstract people can place their hands on it, which may stimulate their abstract mental connections'.

Theme 2a: Freedom to explore

Throughout the coding and thematic analysis of the data strong links between the pedagogical motivation behind the offered activities and how participants perceived the Math Circus were found. The director and producer were motivated by a desire to offer a platform where people could explore mathematical ideas to their own levels of interests without any prerequisites or proof of outcome. The aim was to offer a trial-and-error process that could draw out participants' previous experiences to be linked to the current task to create new understanding. The director and producer see fostering confidence to make errors, spot them, solve, and continue the journey pivotal for mathematics education. The trial and error aspect was both planned and perceived by participants, as a platform to deepen understanding. The researcher experienced the effectiveness of the trial-and-error aspect in the groups she collaborated with. When the groups hit obstacles, there was no judgement but instead a joint enthusiasm to figure out what went wrong and to solve the issue in order to proceed with the construction.

Efforts were made to make appropriate support available but not so that it would interfere with the freedom to explore. Most participants felt that appropriate support was available from facilitators when there was interest to further explore the mathematics concepts. However, some participants mentioned they would have liked to be able to pick up some printed materials but worried that it could take away from the experience and satisfaction of independent discovery and add pressure to learn exactly that. Additionally, the participants found the style of the event collaborative at a peer level instead of teacher-student dynamics, and afterwards they found the process empowering as they felt that they had perceived the overall idea and were confident to take on a deeper investigation should they want to.

The Math Circus pedagogy is indeed quite 'Piagetian' in that it creates an environment that allows intellectual construction of knowledge through free exploration and hands-on trial and error process, similar to what Piaget considered young children's way of learning about their world without it being taught to them specifically or formally evaluated (Papert, 1980, p. 7). The Math Circus is a manmade world but within it, participants were free to play and construct through it different levels of understanding. If they desired to deepen their study, scaffolding support was available from the facilitators who could answer questions or direct towards additional resources. Participants and students of the preludial Math Circus Seminar found the available scaffolding support critical in the trial-and-error aspect of their exploration. The students found that mistakes were welcomed as a part of the learning process and that the available guidance increased their confidence to 'figure it out and continue the work'. By reaching out, it could be argued that participants (and Seminar students) open up for a type of Vygotskian scaffolding support within a Zone of Proximal Development (Doolittle, 2019).

Goodman-Strauss' pedagogy emphasizes playful exploration to create a learning environment where mistakes are not judged but instead the learner develops intuition and confidence to spot and analyze any occurring errors. Again, this links to Piaget's ideas of intuitive mathematical learning but furthermore, Goodman-Strauss stresses that in addition to the playfulness, resources to solve errors, and resilience to acquire necessary fluency must be in place to ensure rigour as well as intuition. Many scholars worry that an artful or qualitative approach may compromise rigour in mathematics whilst others claim that those worries are unfounded as there is more to consider in education than hard facts and truths (Bochner, 2018). This study considers rigour and qualitative learning through the arts such as storytelling and play, to be an interwoven phenomenon, where one supports the other to create a space for deeper levels of understanding.

Within the Math Circus pedagogy lies the aim to create a joyful maths moment that participants could look back to during future mathematical struggles. Goodman-Strauss considers it like planting a seed that may or may not grow. Mathematics anxiety often derives from negative or humiliating experiences in mathematics education and research shows that its consequences include nervousness and avoidance of anything mathematics-related, which then affects motivation, performance, understanding, attitudes, and self-confidence (Ashcraft, 2002). A creative learning environment can significantly reduce mathematics anxiety (Boaler, 2016). Therefore, the playful creativity embedded in the Math Circus could potentially counteract negative mathematics experiences as all our interviewees mentioned how the playfulness demonstrated by the facilitators extended to participants, creating a memorable moment that they imagined they could look back to with joy. Our data indicates that by establishing a basic understanding of an idea through constructive work, confidence to take on further hard work can be cultivated. Our interviewees agreed that seeing the complex abstract materialize before their eyes created a level of comprehension they felt they could use to take on deeper investigation should they desire to do so. One of our interviewees also mentioned how going to similar mathematics festival events as a child fostered her confidence and interest to take on university mathematics as a teenager.

Both the interviewed parents and teachers mentioned how participating in the Math Circus seemed to strengthen their children's/ students' confidence in their abilities to explore and enjoy the mathematics within the constructions. Witnessing adults being excited to play with mathematics also boosted the children's perception of mathematics being creative and fun rather than a boring or complicated chore. The parents mentioned that working with adult mathematics enthusiasts created a feeling of peer collaboration instead of teacher-student relationship. The students felt that they were respected and had a serious role and a voice in the process. The middle school teacher further mentioned how the social dynamics in his class shifted and suddenly students who normally did not have much confidence in their own mathematical abilities were able to work with classmates they normally did not interact with. Furthermore, they were able to discuss the work and start to analyze the mathematical concepts behind the structures.

Theme 2b: Creativity in mathematics education

In addition to the creative thinking necessary to develop how to concretize the abstract into collaborative Math Circus activities, participants also had to apply creativity to construct the objects. The bamboo stars and hyperbolic sculptures were slightly more restrictive than other activities in terms of outcome but treading the path to the final product required analytic and logical thought. Other Math Circus activities were more open in terms of freedom to explore through trial and error. 'Construction play with mathematically structured objects can facilitate children's attention to mathematical properties of objects as they overcome trouble to meet practical goals' (Jasien & Horn, 2018, p. 630). Though based more on collaborative constructions rather than specific already made real objects, the Math Circus activities brought abstract ideas into reality. Once out there, connections to other real-world objects could be made. Weinhandl & Lavicza also found that 'educational technologies and integrating authenticity into the learning environment could further lead to a learning environment having laboratory-like characteristics' (2021, p. 293). The interviewed participants, students and facilitators mentioned how the trial-and-error aspect of the Math Circus and the Signature Seminar simulated science lab protocols.

The Math Circus design can also lower barriers of low confidence or doubts about who belongs in mathematics, as the activities had no specific leader. The participants needed to collaborate to figure out how to proceed, although gentle scaffolding support was available from facilitators when needed. The parents mentioned how their children felt that they had a say in the decision-making and that the overall dynamics of the event were peer-like rather than the conventional teacher-student dynamics of a mathematics classroom. The Math Circus pedagogy can be connected to ideas of how 'math-phobia' 'obstructs mathematical understanding' and controls 'how people view themselves as mathematical learners as deficiency in school mathematics turns into "identity" and learning is transformed from the early child's free exploration of the world to a chore beset by insecurities and self-imposed restrictions' (Papert, 1980, p. 8). The Math Circus layout and materials had several references to participants' potential existing understanding or experiences. Despite not being a 'real' circus the connotations of the name of the Math Circus alone brought on cognitive association of whimsical fun. The participants told us that the name itself sparked their interest enough that they felt they could not miss it.

Theme 2c: Playfulness

The Math Circus attendees approached the different activities in several ways. Some of the activities, such as the chalk tessellations, Curvahedra, and foam strips offered more free exploration than for example the Bamboo Stars and Hyperbolic sculpture which both had a structured end result. Some participants dive straight in and started exploring the tasks in their own way, coming up with different methods, failing and starting over, whilst others analyzed the final product in order to figure out the 'correct' ways to complete the task. The careful analysis was more important in the Bamboo Stars and Hyperbolic sculpture construction than other activities.

In the Math Circus, an invitation to play and 'goof around' with 'serious or advanced' mathematics ideas is extended to anyone who would like to engage. In his work over the decades, Chaim Goodman-Strauss has incorporated play into his mathematics. In his mind, playfulness is the key to learning mathematics and in this context he looks back to a memory from when he first met renown mathematician John Conway[7], later his collaborator, and the two of them (adult mathematicians) ended up spread out on the floor playing with mathematical models completely oblivious of what the other adults in the room might think.

The collaborative aspect of the Math Circus' activities is closely linked to learning theories of play. Through play and drama, students can build the confidence and the skills needed to work with others, and drama may also enhance creativity (Thorkelsdóttir & Jónsdóttir, 2022). Goodman-Strauss' design of the Math Circus layout relates to van Oers' conclusion that mathematical thinking in young children is a cultural process where 'mathematical meaning can be assigned to (spontaneous) actions' of play that can be expanded through 'collaborative problem solving with more knowledgeable others in the context of activities that make sense to the children' (van Oers, 2010, p. 34). The Math Circus activities were mostly only possible to complete through collaborations that made it clear that collaborative creative processes generate a more dynamic learning than working on one's own (Schindler & Lilienthal, 2022)

Theme 2d: Social dynamics and cultural relevance

Mathematical understanding is necessary in modern western societies and performances in mathematics can really influence access to education and employment. In this context, Yeh et al. (2017) refer to mathematics as powerful. Fast scientific and technological shifts call for creativity and problem-solving skills where critical aspects of success include non-judgmental learning environments, connections with lived experiences, and plenty of time to contemplate and construct new ideas (Kettler et al., 2018). Massive reforming efforts to make mathematics education more equitable have been taken on lately but despite that, mathematics education remains inaccessible for too many. Ching and Roberts (2022) argue that these reforms have failed to change the teaching methods which can maintain the barriers. The Math Circus provided a joyful environment without judging anyone for potential errors, plenty of potential links to participants' previous experiences and understanding, as well as a whole day of exploration giving plenty of time for those who wished or happened to become engrossed in the work. Obviously there were no tests or drilling projects but within the process itself as the facilitators pointed out is a path of trial and error where the final product will show whether or not you 'got it' but the path to it was up to each participant.

The middle school teacher's observation of how social dynamics moved during the Math Circus links to Chen and Horn's (2022) call for *critical bifocality* in how education researchers deal with 'oppressive structures and individual agency in reproducing, sustaining, and contesting marginalization' in mathematics education (p.1). This means that researchers should look beyond the most obvious categories of inequity, marginalization, and oppression with a focus on interactions rather than individuals analyzing subtle intersectionality and privilege, scoping beyond the visible to include the invisible contributors to systemic disadvantage and oppression of certain groups. The Math Circus fits within the context of dispersing certain structures of interactions (e.g. within a certain classroom) that can contribute to marginalization or oppression. The Math Circus could be utilized to distribute certain potentially oppressive dynamics of a group in an organic way, as, within a festival style event, social structures and interactions can be restructured from the classroom conventions, and the activities themselves carried links to cultural backgrounds.

The Math Circus learner's agency is high as instead of formal obligations the learning experience is based on collaborations and independent exploration. From these results, it can be concluded that events like the Math Circus can impact cognitive flexibility and the application of critical thinking, as Orakci (2021), and Sachdeva (2019), found to be a result of high learner autonomy. However, our data also supports results showing the importance of adequate support being available within the free play-like exploration to scaffold learning when needed.

Conclusion

The findings of this study indicate that an invitation to explore complex mathematical ideas through interactive, hands-on exploration, through drama and performative storytelling can remove barriers created by the notorious reputation of mathematics being a difficult subject only meant for a few specially chosen to be able to enjoy. The festive elements created by the collaboration, performative aspects, and lack of any testing or formal requirements for success, remove the sense of seriousness often associated with school mathematics where the learner needs to formally prove what they have learned. Despite not requiring formal proof of success the Math Circus construction offers ways to evaluate progress and success as through the trial-and-error process of the construction it becomes clear to the learner whether or not a sufficient comprehension is in place. Furthermore, our findings indicate that through such an interactive event several connections can be made to learners' previous experiences, and social power structures can be rearranged in a positive way to counteract hard to measure inequitable power structures within groups. The performative aspect of the event added elements of joy and playfulness as well as relevance, playfulness, and creativity which can foster new levels of understanding and strengthen critical thinking skills to develop and communicate ideas.

Past and current theories and trends demonstrate different perspectives of what accounts for fruitful mathematics education. Calls for systemic reforms and shifting lenses are common, and regularly a single method dominates the discourse until yet another lens or a successful method takes over. This paper was not at all meant to solve the issues in mathematics education but rather to demonstrate that meaningful exploration of advanced mathematical ideas is possible without building on a specific foundation of prior mathematical learning. Through hands-on engagement and collaboration, gently held together by interactive storytelling, we are invited to keep exploring pathways to the creative beauty of mathematics.

Notes

1. Curvahedra is a construction puzzle system where identical cut pieces link together to build a variety of 3D structures.
2. https://news.uark.edu/articles/59280/experience-the-beauty-of-mathematics-in-courtyard-curvahedra-
3. A rhombic triacontahedron is a polyhedron with 30 rhombic faces, and icosahedral symmetry.
4. Penrose tilings are aperiodic tilings that can have both reflection symmetry and fivefold rotational symmetry (Gardner, 1977). They can infinitely tile a plane by copies of a pair of certain shapes (most popularly kites and darts) (Penrose, 1979). Penrose tilings' properties and applications have been studied in fields beyond mathematics, including physics, material science, and art. For further information, see the works of John H. Conway (b.1 937- d. 2020) (Radin, 2021; Schattschneider, 2021); de Bruijn (1981a; 1981b), Shechtman et al. (1984), Levine and Steinhardt (1984), Lu and Steinhardt (2007); Sakai and Arita (2019); Yan et al. (2020); and Padilla (2022).
5. Islamic geometric tiling patterns are ancient forms of islamic decorations usually consisting of repetitions of overlapping and interlaced, both basic and more complex, geometric shapes which are arranged by different techniques, including rotations, symmetries, and regular and semi-regular tessellations of shapes that can be repeated infinitely to cover the surface without any gaps (Bonner, 2017). The art form is found throughout the Middle East, North Africa, Europe, and beyond, including Alhambra in Spain, Golestan Palace in Tehran, and Taj Mahal in India. Bonner's book *Islamic Geometric Patterns* (2017) is a great resource for further study of this rich and versatile mathematical art form.
6. https://aplus.uark.edu/courtyard-curvahedra/
7. https://www.princeton.edu/news/2020/04/14/mathematician-john-horton-conway-magical-genius-known-inventing-game-life-dies-age, https://dof.princeton.edu/about/clerk-faculty/emeritus/john-horton-conway

Acknowledgements

The researchers would like to thank the entire Math Circus team and interviewees who generously devoted their time to be interviewed. We would also like to thank the anonymous reviewers for their kind and helpful suggestions.

Disclosure statement

No potential conflict of interest was reported by the authors.

Declaration of interest statement

No funding was received for this study, but the corresponding researcher would like to declare familial ties to Edmund Harrriss, the founder of Curvahedra, and a member of the editorial team for the Journal of Mathematics and the Arts (Special issue The Art of Mathematical Illustration; and Focus). Harriss was part of the Math Circus offerings but was not part of this study otherwise.

Ethics

The research complies with the University of Iceland's guidelines for ethical research.

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